MCS 251-MT1 (Summer)

Q1. Let the sets $A = \{(x,y) \in \mathbb{R}^2 : y > x^2 - 1\} \cup \{(x,y) \in \mathbb{R}^2 : y < 1 - x^2\}$ and $B = \{(x,y) \in \mathbb{R}^2 : x \in \mathbb{Z}, y \in \mathbb{Q}\}$ be given. Find;
a) $Int(A), cl(A), \partial A$ b) $Int(B), cl(B), \partial B$.
Describe your answer.

Q2. Construct sets to show that each of the following is **False**:
a) $A \subseteq B \Rightarrow \partial A \subseteq \partial B$
b) $\partial S = \partial(IntS)$
c) $IntA \cup IntB = Int(A \cup B)$

Q3. Show that $f(x) = \sqrt{x}$ is uniformly continuous on $[0, \infty)$.

Q4. Let $f_n : [0,\infty) \to \mathbb{R}, f_n(x) = nxe^{-n^2x}$.
a) Is $f_n \to 0$ pointwise on $[0,\infty)$?
b) Is $f_n \to 0$ uniformly on $[0,\infty)$?
Describe your answer.

Q5. Show that $\sum_{n=0}^{\infty} e^{-nx}\cos(nx)$ converges uniformly on $[1,\infty)$.

Q6 (Bonus). Prove that there is no continuous map from $[0,1]$ onto $(0,1)$.